Appendix C

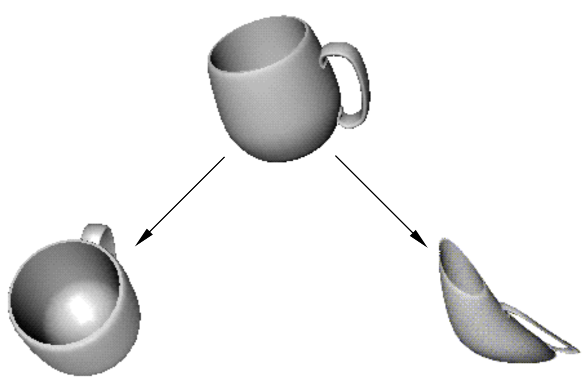

Figure C.1: Transformation and deformation. Although the object on the right certainly looks deformed, it is still easily recognized as related to the object on the left. In some sense, therefore, its shape is preserved.

In some cases, it may be desirable to define shape up to a group of transformations that is less restrictive than the orthogonal group, or, in other words, to allow *deformation* (see Figure C.1).

Consider Dali's *Persistence of Memory*: our perception of the thing suspended from the tree branch as a deformed clock rather than an uninterpretable shape attests to the perceptual equivalence of some shapes related by deformations rather than transformations. In that case, a suitable framework for the definition of a shape space can be provided by the theory of Riemann surfaces (Krushkal', 1979). Consider surfaces (shapes) of a given genus, that is, simply-connected (one-part) surfaces with the same number of holes in them (sphere and all its "rubber" deformations are surfaces of genus 0; the doughnut and the cup — surfaces of genus 1, etc.). Two such shapes related by a *conformal* mapping can be considered equivalent (belonging to the same class), with a *quasiconformal* mapping taking one shape class into another. The resulting shape space (known as the Teichmüller space) possesses a natural Riemannian metric, in which distance between two shapes is defined by the deviation from conformality of the quasiconformal mapping by which they are related (Krushkal', 1979). The Teichmüller space can be parameterized by a small set of real numbers, which provide a possible coordinate system for the resulting shape space (Sundararaman, 1980).

In Teichmüller theory, the parameterization of Riemann surfaces in a given equivalence class of (conformal) deformations is unique. This follows from the solution to the famous Problem of Moduli in algebraic geometry (Sundararaman, 1980).