Within two years of the appearance of these lines Hilbert's fifth problem had been solved — by three American-born, American-trained mathematicians [1], [3].

"A well-written Life," said Carlyle, "is almost as rare as a well-spent one." *The Apprenticeship of a Mathematician* is the well-written account, *volumen non illepidum neque invenustum*, of (half) a life well-spent. Dare we hope for a sequel?

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The Illusion of Reality
by Howard Resnikoff

New York: Springer-Verlag, 1989. viii + 339 pp.
US $54.00, ISBN 0-387-96398-7

The Geometry of Vision
edited by Robert Melter, Azriel Rosenfeld, and Prabir Bhattacharya

Providence, RI: American Mathematical Society, 1991.
viii + 237 pp.
US $90.00, ISBN 0-8218-5125-X

Visual Structures and Integrated Functions
edited by Michael Arbib and Jörg-Peter Ewert

New York: Springer-Verlag, 1991. xii + 441 pp.
US $49.00, ISBN 0-387-54241-8

Reviewed by Shimon Edelman

The first steps of a person who embarks on a study of visual perception are difficult for many of the same reasons that we find it difficult to imagine how our mother tongue sounds to someone who does not understand it, and what it would be like for someone to learn it. We are so loath to acknowledge the difficulty of learning our own language that in speaking to a foreigner we tend to raise our voice, as if unconsciously assuming that we are not understood merely because we are not heard clearly. In this case, as well as in thinking about vision, the first obstacle to be overcome is the conviction that what seems so easy and natural cannot be too complicated. Such misconceptions regarding everyday linguistics can be easily cured by trying to learn a foreign language.

In the study of vision, a good way to appreciate the difficulties involved in seeing is to try to program a computer to see. It becomes apparent that making a computer see is harder than making it solve other information-processing tasks, some of which were once thought to epitomize intelligence (for example, playing chess at the level of a grand master). This realization is apt to bring with it a profound sense of wonder at the performance of our visual systems. Our eyes provide us with a stable and reliable impression of the surrounding world, and do so in the face of the ever-changing illumination and vantage point, and despite the limitations of the available information-processing units, each of which is slow and unreliable. As Marvin Minsky once remarked, the reliability of our visual systems is constantly tested throughout our lifetimes. The degree to which they stand up is manifest in the extreme rarity of situations in which silly behavior, such as an attempt to walk through a window, stems from a failure of vision. It is ironic that vision, so seldom fooled in real-world situations, is so easily fooled by specially contrived stimulation such as that supplied by the well-known visual illusions. This credulity of vision prompts one to consider the possibility that the exquisite tapestry it weaves is nothing but an *illusion of reality*, maintained, perhaps, for the sake of inner peace, and sufficiently reliable for normal behavior, but surprisingly amenable to manipulation. If true, this possibility could have serious implications for understanding the way vision and other perceptual faculties work. Most importantly, it could mean that the representation of the world built by the perceptual systems is not nearly so complete and veridical as the current central dogma of cognitive science would have it.

Contrary to my expectations when I first glanced at the title of *The Illusion of Reality*, Howard Resnikoff's book does not go so far as to question the basic tenets of the state-of-the-art theories of vision. The book is aimed at developing a unifying approach to several problems of measurement and computation. It starts by considering simple problems, such as measuring the length of a rod with a yardstick or calculating successive digits in the decimal expansion of $\sqrt{2}$, and progresses over the course of four chapters to the considerably more complicated problem of understanding how the measurements performed by the human visual system on its input can produce many of the known visual illusions. The reader is well-prepared for (or, at least, forewarned of) the eclectic style of the book by the introductory chapter in which

Resnikoff sets out his framework for information science that brings together the disciplines of thermodynamics, psychophysics (or the study of sensory information processing), communication engineering, and computability. The sheer courage of attempting to review the history of all those disciplines together in 20 pages, which Resnikoff does in search of common threads that are to bind them into an integrated information science, is admirable and should mitigate any complaints of omissions or bias.

Nevertheless, fair play requires that at least some of the reviewed material be put into a proper perspective. It happens that heuristics (see Judea Pearl's book bearing this title [1]) is a well-established branch of computer science and not merely "a term to name neatly an essential process which is still largely a mystery," as Resnikoff would have it. Similar instances of missing essential references occur in other parts of the book. Chapters 2 and 3 are devoted to the mathematics and the physics of information measurement, respectively. A nonphysicist, whose last encounter with Heisenberg's uncertainty principle probably occurred in undergraduate physics, will surely appreciate Resnikoff's lucid treatment of uncertainty in measurement. Chapter 4 rounds off the programmatic first half of the book with an overview of principles of information-processing systems: hierarchy, optimization of information, signals and modulation, and sampling.

Having outlined his framework for information science in the first half of the book, Resnikoff spends the last two chapters applying the newly expounded general principles to an analysis of biological information processing. Most of this part of the book deals with vision; it is here that I felt the most uneasy about Resnikoff's presentation. Some of the biological details found in Chapter 5 are wrong. Retinal rod cells are not insensitive to the spectral composition of light. In fact, their spectral selectivity is similar to that of the cones. The reduced color perception in low-light (scotopic) conditions when only the rods are active is due to there being only one type of rod (compared to three types of cones whose differing peak spectral tuning make the perception of color possible, despite poor selectivity of each individual cone type). Another error appears in the discussion of eye movements, where saccades (fast voluntary "jumps" of the fixation point) are confused with tremor (small irregular involuntary oscillations of the eye). Some of the details are simply irrelevant: What does the chemical structure of rhodopsin have to do with the topic of the book? Perhaps a more problematic thing about Resnikoff's attempt to explain perception by appealing to the principles of information theory is that so little understanding seems to be gained as a result. For example, the explanations of various visual illusions offered in Chapter 5, clearly designed to be the culmination of the book, are disturbingly circular:

...making a computer see is harder than making it solve other information-processing tasks, some of which were once thought to epitomize intelligence...

> It remains to ask why the vision system systematically distorts angular measurements. In the cases discussed, it appears that the quantity of information obtained from an observation tends to be reduced by the illusory effect. This can be interpreted as a mechanism for omitting information that was present in the original scene ... [p. 230]

I feel that such a statement adds nothing to the neurophysiological account of the illusions, offered immediately prior to it in Chapter 5. Unless a good computational reason is found for the omission of information, it is better to assume that it is a by-product of the neural implementation of some other, intrinsically significant, perceptual function. The above example is symptomatic of more general problems that beset attempts to apply information theory to perception. The initial flood of publications that followed the discovery of information theory by psychologists in the fifties dried up a decade later, with review papers bearing titles such as "Information theory and figure perception: the metaphor that failed" [2]. The abuse of information theory in popularizing genetics has been pointed out recently in the *Mathematical Intelligencer* by Jack Cohen and Ian Stewart [3]. Space limitations prevent me from reproducing here the issues raised by these and other critics of the information-theoretic approach. The main argument, however, can be stated in just one sentence: There are certain formal prerequisites for a valid application of information-theoretic tools. Two of these are the knowledge of the representations involved in a communication or computation process and the knowledge of prior probabilities of the various events in the system. Resnikoff's treatment of perception, for example, presupposes that its final product is a representation that is, in a sense, a reconstruction of the world. For an entertaining and provocative discussion of why this may not be true, I refer the reader to a recent book by Daniel Dennett [4].

If the thesis of Resnikoff's book is taken to be a unified approach to vision and other information-processing activities, *The Geometry of Vision*, edited by Melter, Rosenfeld, and Bhattacharya, can be considered its Hegelian antithesis, in form if not in content. This book presents a fragmented picture of the state of the art of digital geometry, one of the many computational aspects of vision. The fragmentation is due to the absence of any form of editorial integration of the 14 papers included in the collection, which were presented at a special session of a meeting of the American Mathematical Society. For example, the reader will find there a paper that advocates a general approach to shape representation based on Minkowski operators of addition and decomposition, and, three chapters later, another paper that proves certain problems of Minkowski decomposition to be intractable (NP-complete). The collection includes two reviews (on digital metrics and on applications of computational geometry in computer vision) and a diverse set of papers on digital straight lines, properties of polygons, and graph connectivity.

The reviews (A Survey of Digital Metrics by Melter and Computational Geometry and Computer Vision by Toussaint) are informative and include ample bibliography. Of the other papers, I found two to be especially good reading. Self-Similarity Properties of Digitized Straight Lines by Bruckstein presents several properties of chain codes (which are discrete representations of contours in binary images) that express the invariance of the digital straightness property over different possible regular subgrids embedded in the integer lattice. Bruckstein's treatment is both comprehensive and definitive: He shows that his approach covers all possible types of chain-code self-similarity. The other paper, Galleries and Light Matchings: Fat Cooperative Guards by Czyzowicz, Rival, and Urrutia, gives a good example of a typical problem in computational geometry ("how many guards are necessary, and how many are sufficient, to patrol the n paintings of an art gallery?"), and includes an elegant solution, based on a reduction to graph coloring. For those who cannot wait until they get a copy of the book to have the answer, here it is: $\lceil 3n + 4 \rceil$ guards are sometimes necessary and always sufficient.

Considering both perception and action within the same framework makes it easier to accept the possibility that complex behavior need not rely on intricate representations of the world . . .

If you take dialectics seriously, you probably half anticipate my labeling the third book, *Visual Structures and Integrated Functions* edited by Michael Arbib and Jörg-Peter Ewert, as a *synthesis* of a wide variety of topics in biological information processing. Indeed, this highly informative collection provides no less, illustrating how the notion of the nervous system as an information processor can be given explanatory power by considering its various functions in the wider context of the organism's behavior. Considering both perception and action within the same framework makes it easier to accept the possibility that complex behavior need not rely on intricate representations of the world, an idea that many students of vision still reject out of hand.

An example of the integrated approach, and one of the central topics of the book, is a set of computational models which together comprise *Rana computatrix*, the "frog that computes." A survey of the development of *Rana computatrix* over the last decade, by Michael Arbib, opens the collection. Arbib's article provides enough of a background even for a newcomer to the field to enjoy the book, and ends with the bold claim that "... the roots of our intelligence in visuomotor coordination point the way to a theory of higher mental functions based on this new paradigm. *Ex Rana computatrix ad omnia.*" The new paradigm to which Arbib refers is the integrated use of schema theory and neural modeling. The concept of a schema in various disciplines of cognitive science (such as neurobiology or artificial intelligence) usually denotes a behavior or a mechanism that subserves a well-defined and limited function (perceptual or motor) and is activated by a certain pattern of values of cognitive variables. Prominent examples of theories of mind that rely on schemalike building blocks are Minsky's *The Society of Mind* [5] and the work by Dennett mentioned above [4]. The second article in the collection, A Prospectus for the Fruitful Interaction Between Neuroethology and Neural Engineering by Jörg-Peter Ewert, complements Arbib's opening overview by considering several concrete examples of behavioral tasks, the neural mechanisms that support these tasks (in real frogs), and the questions they raise for computational modelers.

The rest of the book is divided into five sections (From the Retina to the Brain; Approach and Avoidance; Generating Motor Trajectories; From Tectum to Forebrain; Development, Modulation, Learning and Habituation) and contains a satisfyingly high proportion of interesting articles. It should be noted that both the range of organisms (from toads via zebra finches to macaque monkeys) and the range of behaviors (from fly-catching via bird-song to visual face recognition) that appear in the book are wide enough to vindicate at least to some extent Arbib's claim of the generality of his approach, expressed in the Preface. The frog may well one day turn into a prince and help us see through the illusion of reality.

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